# The surprising maths that explains why coincidences are so common

From repeat lightning strikes to identical lottery draws, mathematician Sarah Hart explains why incredibly unlikely events happen all the time



Eerie events often have mundane explanations

I had a strange experience in a cafe recently. When paying

for coffee, I asked to use the toilet. After I tapped my PIN into the card reader, the barista handed me a slip of paper that was blank apart from a single number – the same one I had just tapped into the keypad! It turned out that this was also the entry code for the bathroom door.

Was this evidence of a shadowy plot by the agents of Big Caffeine? No, it was just a fluke, but the event got me thinking about coincidences more generally. Occasionally, we are faced with events that seem so improbable we can't help but feel the universe is sending us a message. As a mathematician, I know that coincidences are often far more likely to occur than we think – and this fact can have serious consequences everywhere from the science lab to lottery kiosks and the law courts.

[Mathematicians have discovered a mind-blowing new kind of infinity](#)

[It may sound strange, but mathematicians have created an entire ladder of infinities, each larger than the next. Now a new kind of infinity threatens to upset that order, and perhaps redefine the structure of the mathematical universe](#)

My coffee shop experience is a good place to start because we can calculate the exact probability of it happening by chance. There are 10,000 possible four-digit numbers, meaning a 1 in 10,000 probability that the door code matches my PIN. Unlikely, yes, but let us put it in context. The cafe was in York, UK – a popular tourist destination with

8.9 million visitors annually. If each visitor has one bank card, we would expect around 890 cards each year to have the same PIN as that cafe's bathroom door code. I don't feel so special now.

## The 0.7 rule

The cafe coincidence illustrates an important fact: even very improbable events are likely to occur if given enough opportunity. We can express this mathematically as follows. With $k$ opportunities for an event to occur, at each of which there is a one in $n$ chance of it actually happening, then (assuming the events are independent) the probability it occurs at least once is:

$$1 - (1 - 1/n)^k$$

Don't let the algebra put you off: the take-home message is that the chance of a coincidence rises quickly as the number of opportunities increases. From this formula, we can derive the following fact: the probability of a "one-in-$n$" event happening at least once exceeds 50 per cent whenever there are more than $0.7n$ independent opportunities for it to occur – a phenomenon I call the 0.7 rule.

[Stephen Hawking's final theorem turns time and causality inside out](#)

Let us consider a couple of examples. Suppose everyone in Birmingham has a one-in-a-million chance each night of dreaming tomorrow's news. As the population is over 700,000, it is more likely than not that each morning someone in Birmingham becomes convinced they are psychic.

Every day, there are many ways a freak coincidence could happen to you. Say you have one opportunity per hour to experience such an event, and the median life expectancy of 706,000 hours, there is a better than even chance you will experience a one-in-a-million coincidence at least once.

Such analyses can help explain some of history's most breathtaking coincidences, such as the curious case of the Bulgarian national lottery. In September 2009, the same numbers – 4, 15, 23, 24, 35 and 42 – came out in two consecutive draws.

The human brain can easily attribute meaning to random occurrences

It sounds like a fix, but the statistician [David Hand](#) calculated that the game would only have to run for 43 years before it becomes more likely than not for the same numbers to be drawn at least twice. (The lottery had, by this point, [been running for 52 years](#).)

*Roy Sullivan was struck by lightning seven times*

Each lottery ticket has an equal chance of winning, but people aren't lottery tickets, and so the probability of experiencing a rare event can vary from individual to individual. [Roy Sullivan](#), a Virginia park ranger, was struck by

lightning seven times between 1942 and 1977. That is certainly unlucky, but he had more chance than most. There is more lightning in warmer, wetter parts of the US, and people with an outdoor job are more exposed than those who are protected indoors. This latter risk factor partly explains why men account for 84 per cent of deaths by lightning strike.

[How mathematics can help you divide anything up fairly](#)

Occasionally, odds can appear more impressive than they truly are. Famous "psychic" [Jeane Dixon's prediction](#) that John F. Kennedy would win the 1960 presidential election and die in office looks unsettlingly prescient, until I tell you that she had also forecast a Nixon victory in 1960 and the start of World War III in 1958. Never mind, Jeane. As Niels Bohr famously said, prediction is very hard, especially of the future.

The laws of probability show that rare events happen surprisingly often given enough opportunity

Jonathan Higbee

Sometimes, of course, there really is something more going on. If [Edward Jenner](#) had dismissed the curious fact that milkmaids were unusually immune to smallpox, he might never have uncovered the protection provided by a previous cowpox infection, which led to the world's first successful vaccines.

## Detecting plagiarism

Distinguishing between coincidences and deeper causes is central to the scientific method and can also resolve

accusations of plagiarism. Ed Sheeran's chart-topper *Shape of You*, for instance, was the subject of a 2022 lawsuit claiming that a sequence of four notes was lifted from Sami Chokri's 2015 song *Oh Why?*

"Coincidence is bound to happen if 60,000 songs are being released every day on Spotify," [Sheeran told his fans](#) after winning the lawsuit. "That's 22 million songs a year and there's only 12 notes available."

He is right – with 12 notes in the Western octave, the number of possible four-note phrases is a mere 20,736. It is a mathematical certainty that many of those 22 million songs will contain uncannily similar passages.

In academia, distinguishing coincidental resemblance from plagiarism or AI assistance in student essays is a growing challenge. One plagiarism detection tool called Turnitin has an estimated false positive rate of 1 per cent for identifying documents with more than 20 per cent AI content. But if you are in that 1 per cent, how do you prove you haven't cheated? One University of California, Davis, student, Louise Stivers, [was only able to convince the university of her innocence](#) by providing time-stamped drafts of her work.

Serendipity may be a source of beauty

Denis Cherim

At this time of year, we often find ourselves chatting to strangers at parties, looking for things in common. Happily, such connections almost certainly exist. It has been estimated that we each know 1000 people (acquaintances, colleagues, neighbours: I will just say "friends" for short). If these connections are randomly distributed, it turns out there is a better than 99 per cent chance that you and I either have a mutual friend or a friend of yours knows a friend of mine.

# Small worlds

In real life, social networks are far from randomly distributed: you are more likely to know someone from your own city than someone from elsewhere, for instance. Surprisingly, however, this doesn't have much impact on these effects.

Mathematicians [Steven Strogatz and Duncan Watts](#) discovered that the presence of a few highly connected individuals can lead the network's overall distribution to approximate that of a random one. This means that, even if people only have 100 friends on average, any two people in the world are probably linked by a chain of fewer than five mutual friends.

The cliché of dinner party small talk turns out to be true: it really is a small world. Thanks to the laws of mathematics, coincidences abound. Events that seem to be a matter of fate are often little more than a numbers game.

*Sarah Hart is a mathematician and author of Once Upon a Prime: The wondrous connections between mathematics and literature*